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## REMARKS BY TRACY A. PIERCE, Harvard University.

It is well known that every prime of the form  $4m + 1$  is the sum of two squares. Using Lucas's form of an odd perfect number (see Problem 211 above) we see that

$$(4m + 1)^{4k+1}n^2 = (x^2 + y^2)[(4m + 1)^{2k}]^2n^2 = (x^2 + y^2)N^2 = X^2 + Y^2,$$

contrary to the proposition as proposed.

## 229. (March, 1915.) Proposed by WALTER C. EILLS, Whitman College.

If  $p$  and  $q$  are integers and  $p$  is prime and positive, find the condition on  $q$  that the equation  $p^x = qx$  shall have integral solutions, solve for  $x$ , and show that for a special value of  $p$  it has two solutions for a certain  $q$ , otherwise only one.

## I. SOLUTION BY FRANK IRWIN, University of California.

Since  $p$  is prime, we must have  $x = p^a$ ,  $q = p^b$ , where  $a$  and  $b$  are positive integers or zero (for  $a$ ). Now  $p^{a+b} = qx = p^{p^a}$ , so that  $b = p^a - a$ , and  $q$  is necessarily of the form  $p^{(p^a-a)}$ . This condition is evidently also sufficient.

Given, then, such a  $q$ , the exponent  $p^a - a$  may be determined; then  $a$ , which will give us  $x$ , is that number which must be added to this exponent to make it equal to the *next higher* power of  $p$ . For no power of  $p$  can lie between  $p^a - a$  and  $p^a$ , since  $p^a - a > p^{a-1}$ , as may be readily proved, for instance by mathematical induction.

Of the cases that require special investigation,  $a = 0$ , and  $p = 2$  with  $a = 1$  or 2, the only one for which, given  $p^a - a$ , there is more than one solution for  $a$ , is the case

$$2^a - a = 1,$$

which has two solutions  $a = 0, 1$ . There are two solutions of our problem then for the case  $p = 2$ ,  $q = 2$ , viz.,  $x = 1, 2$ .

## II. SOLUTION BY THE PROPOSER.

Consider the two functions,  $y = p^x$ ,  $y = qx$ . For  $x = k$  (any integer),  $y = p^k$ . The slope of line  $y = qx$ , passing through  $(k, p^k)$ , is  $q = p^k/k$ , which is integral if and only if

$$k = p^n \quad (n = 0, 1, 2, 3, \dots), \text{ i. e., } q = p^{(p^n-n)}.$$

(If  $k < 0$ ,  $q$  is fractional since it is then  $= 1/kp^k$ .)

Substituting this value of  $q$  in the given equation, it is easily seen that it is satisfied if and only if

$$x = p^n \quad (n = 0, 1, 2, 3, \dots).$$

Consider the exponent of  $p$ , namely  $p^n - n$ . We have

$$[p^n - n]_{n=0} = 1, \text{ and } [p^n - n]_{n=1} = p - 1.$$

Then  $x_1 = p^0$  and  $x_2 = p^1$  will be solutions of the given equation if  $1 = p - 1$ , i. e., if  $p = 2$ . From the graphs of the exponential function it is easily seen that  $y = qx$  can have but one integral intersection if  $p \neq 2$ ,  $n_1 \neq 0$ ,  $n_2 \neq 1$ .

The equation having two solutions is  $2^x = 2x$ , of which  $x_1 = 1$ ,  $x_2 = 2$ .

## 230. (April, 1915.) Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Find three numbers such that their sum, the sum of their squares, and the sum of their cubes, shall be a cube.

*Note.*—W. D. Cairns says this problem, which was proposed in *L'Intermediaire* in 1900, remains unsolved to date, even though it was reprinted in February, 1913.

## REMARKS BY ARTEMAS MARTIN, Washington, D. C.

The above problem was published in the *Mathematical Visitor*, Vol. I, No. 1 (Erie, Pa., March, 1877), page 6, as No. 9 in a list of "Unsolved Problems." So far as the writer at present knows that was the first publication of the problem and it still remains "unsolved."